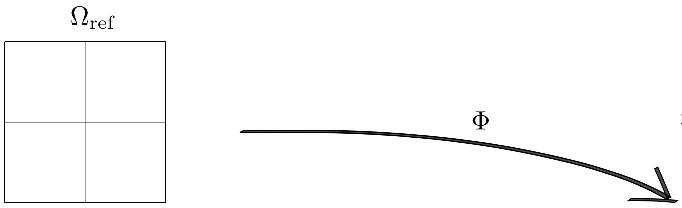

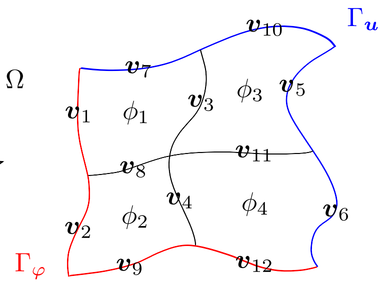

∗ To make the illustration clear, we use a 2D plot to represent the domain.

▶ $\boldsymbol{v}_1$, $\boldsymbol{v}_1$, $\cdots$, $\boldsymbol{v}_{12}$: basis face polynomials

▶ ϕ_1, ϕ_2, ϕ_3, ϕ_4: basis volume polynomials

∗ `Discrete system before applying the boundary conditions:`

$$
\begin{matrix} v_1 \to \\ v_2 \to \\ v_3 \to \\ v_4 \to \\ v_5 \to \\ v_6 \to \\ v_7 \to \\ v_8 \to \\ v_9 \to \\ v_{10} \to \\ v_{11} \to \\ v_{12} \to \\ \phi_1 \to \\ \phi_2 \to \\ \phi_3 \to \\ \phi_4 \to \end{matrix}
\begin{bmatrix}
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
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\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & & & & \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & & & &
\end{bmatrix}
\begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \\ \underline{u}_4 \\ \underline{u}_5 \\ \underline{u}_6 \\ \underline{u}_7 \\ \underline{u}_8 \\ \underline{u}_9 \\ \underline{u}_{10} \\ \underline{u}_{11} \\ \underline{u}_{12} \\ \underline{\varphi}_1 \\ \underline{\varphi}_2 \\ \underline{\varphi}_3 \\ \underline{\varphi}_4 \end{bmatrix}
=
\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ b_8 \\ b_9 \\ b_{10} \\ b_{11} \\ b_{12} \\ \circ \\ \circ \\ \circ \\ \circ \end{bmatrix}
$$

- For $i \in \{3,4,8,11\}$ (internal basis face polynomials): $\boldsymbol{v}_i \cdot \boldsymbol{n} = 0$, therefore, $b_i = 0$.
- For $i \in \{1,2,9,12\}$ (basis face polynomials on Γ_φ): $b_i = \int_{\partial\Omega} \widehat{\varphi}\,(\boldsymbol{v}_i \cdot \boldsymbol{n})\,\mathrm{d}\Gamma = \widehat{\varphi}_i$.
- For $i \in \{5,6,7,10\}$ (basis face polynomials on $\Gamma_{\boldsymbol{u}}$): to set $\underline{u}_i = \int_{F_i} \widehat{u}\mathrm{d}\Gamma = \widehat{u}_i$.

∗ `Fully discrete system after applying the boundary conditions:`

$$
\begin{matrix} v_1 \to \\ v_2 \to \\ v_3 \to \\ v_4 \to \\ v_5 \to \\ v_6 \to \\ v_7 \to \\ v_8 \to \\ v_9 \to \\ v_{10} \to \\ v_{11} \to \\ v_{12} \to \\ \phi_1 \to \\ \phi_2 \to \\ \phi_3 \to \\ \phi_4 \to \end{matrix}
\begin{bmatrix}
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
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\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
 & & & & 1 & & & & & & & & & & & \\
 & & & & & 1 & & & & & & & & & & \\
 & & & & & & 1 & & & & & & & & & \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
 & & & & & & & & & 1 & & & & & & \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
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\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & & & &
\end{bmatrix}
\begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \\ \underline{u}_4 \\ \underline{u}_5 \\ \underline{u}_6 \\ \underline{u}_7 \\ \underline{u}_8 \\ \underline{u}_9 \\ \underline{u}_{10} \\ \underline{u}_{11} \\ \underline{u}_{12} \\ \underline{\varphi}_1 \\ \underline{\varphi}_2 \\ \underline{\varphi}_3 \\ \underline{\varphi}_4 \end{bmatrix}
=
\begin{bmatrix} \widehat{\varphi}_1 \\ \widehat{\varphi}_2 \\ 0 \\ 0 \\ \widehat{u}_5 \\ \widehat{u}_6 \\ \widehat{u}_7 \\ 0 \\ \widehat{\varphi}_9 \\ \widehat{u}_{10} \\ 0 \\ \widehat{\varphi}_{12} \\ \circ \\ \circ \\ \circ \\ \circ \end{bmatrix}
$$

∗ `We can also eliminate rows and columns 5, 6, 7 and 10 to obtain a smaller system:`

$$
\begin{matrix} v_1 \to \\ v_2 \to \\ v_3 \to \\ v_4 \to \\ v_8 \to \\ v_9 \to \\ v_{11} \to \\ v_{12} \to \\ \phi_1 \to \\ \phi_2 \to \\ \phi_3 \to \\ \phi_4 \to \end{matrix}
\begin{bmatrix}
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
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\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
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\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & & & & \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & & & & \\
\circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & & & &
\end{bmatrix}
\begin{bmatrix} \underline{u}_1 \\ \underline{u}_2 \\ \underline{u}_3 \\ \underline{u}_4 \\ \underline{u}_8 \\ \underline{u}_9 \\ \underline{u}_{11} \\ \underline{u}_{12} \\ \underline{\varphi}_1 \\ \underline{\varphi}_2 \\ \underline{\varphi}_3 \\ \underline{\varphi}_4 \end{bmatrix}
=
\begin{bmatrix} \widehat{\varphi}_1 \\ \widehat{\varphi}_2 \\ 0 \\ 0 \\ 0 \\ \widehat{\varphi}_9 \\ 0 \\ \widehat{\varphi}_{12} \\ \circ \\ \circ \\ \circ \\ \circ \end{bmatrix}
- \widehat{u}_5 \begin{bmatrix} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{bmatrix}
- \widehat{u}_6 \begin{bmatrix} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{bmatrix}
- \widehat{u}_7 \begin{bmatrix} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{bmatrix}
- \widehat{u}_{10} \begin{bmatrix} \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{bmatrix}
$$