Mass matrices

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1 Mass matrix $\mathsf{M}_\mathbf{N}$

M_N is the mass matrix of space $\mathrm{NP}_N(\Omega)$, and its entries are

$$\begin{aligned}&\mathsf{M}_\mathrm{N}|_{i+1+j(N+1)+k(N+1)^2,\ l+1+l(m+1)+n(N+1)^2}\\ &\quad=\int_{\Omega_\mathrm{ref}}\sqrt{g}\ \mathrm{lll}^{i,j,k}\mathrm{lll}^{l,m,n}\mathrm{d}\Omega,\quad i,j,k,l,m,n\in\{0,1,\cdots,N\}\,.\end{aligned}$$

2 Mass matrix $\mathsf{M}_\mathbf{E}$

M_E is the mass matrix of space $\mathrm{EP}_{N-1}(\Omega)$. It can be written block-wise as

$$\mathsf{M}_\mathrm{E}=\begin{bmatrix}M^{1,1} & M^{1,2} & M^{1,3}\\ M^{2,1} & M^{2,2} & M^{2,3}\\ M^{3,1} & M^{3,2} & M^{3,3}\end{bmatrix},$$

where

$$\begin{aligned}
&M^{1,1}\big|_{i+jN+kN(N+1),\ l+mN+nN(N+1)}\\
&\quad=\int_{\Omega_\mathrm{ref}}\sqrt{g}\ g^{1,1}\ \mathrm{ell}^{i,j,k}\mathrm{ell}^{l,m,n}\mathrm{d}\Omega,\quad i,l\in\{1,2,\cdots,N\}\,,\ j,k,m,n\in\{0,1,\cdots,N\}\,,\\
&M^{2,2}\big|_{i+1+(j-1)(N+1)+kN(N+1),\ l+1+(m-1)(N+1)+nN(N+1)}\\
&\quad=\int_{\Omega_\mathrm{ref}}\sqrt{g}\ g^{2,2}\ \mathrm{lel}^{i,j,k}\mathrm{lel}^{l,m,n}\mathrm{d}\Omega,\quad j,m\in\{1,2,\cdots,N\}\,,\ i,k,l,n\in\{0,1,\cdots,N\}\,,\\
&M^{3,3}\big|_{i+1+j(N+1)+(k-1)(N+1)^2,\ l+1+m(N+1)+(n-1)(N+1)^2}\\
&\quad=\int_{\Omega_\mathrm{ref}}\sqrt{g}\ g^{3,3}\ \mathrm{lle}^{i,j,k}\mathrm{lle}^{l,m,n}\mathrm{d}\Omega,\quad k,n\in\{1,2,\cdots,N\}\,,\ i,j,l,m\in\{0,1,\cdots,N\}\,,\\
&M^{1,2}\big|_{i+jN+kN(N+1),\ l+1+(m-1)(N+1)+nN(N+1)}\\
&\quad=\int_{\Omega_\mathrm{ref}}\sqrt{g}\ g^{1,2}\ \mathrm{ell}^{i,j,k}\mathrm{lel}^{l,m,n}\mathrm{d}\Omega,\quad i,m\in\{1,2,\cdots,N\}\,,\ j,k,l,n\in\{0,1,\cdots,N\}\,,\\
&M^{1,3}\big|_{i+jN+kN(N+1),\ l+1+m(N+1)+(n-1)(N+1)^2}\\
&\quad=\int_{\Omega_\mathrm{ref}}\sqrt{g}\ g^{1,3}\ \mathrm{ell}^{i,j,k}\mathrm{lle}^{l,m,n}\mathrm{d}\Omega,\quad i,n\in\{1,2,\cdots,N\}\,,\ j,k,l,m\in\{0,1,\cdots,N\}\,,\\
&M^{2,3}\big|_{i+1+(j-1)(N+1)+kN(N+1),\ l+1+m(N+1)+(n-1)(N+1)^2}\\
&\quad=\int_{\Omega_\mathrm{ref}}\sqrt{g}\ g^{2,3}\ \mathrm{lel}^{i,j,k}\mathrm{lle}^{l,m,n}\mathrm{d}\Omega,\quad j,n\in\{1,2,\cdots,N\}\,,\ i,k,l,m\in\{0,1,\cdots,N\}\,,
\end{aligned}$$

and $M^{2,1}=\left(M^{1,2}\right)^\mathsf{T}$, $M^{3,1}=\left(M^{1,3}\right)^\mathsf{T}$, $M^{3,2}=\left(M^{2,3}\right)^\mathsf{T}$.

3 Mass matrix $\mathsf{M}_{\mathbf{F}}$

M_{F} is the mass matrix of space $\mathrm{FP}_{N-1}(\Omega)$. It can be written block-wise as

$$\mathsf{M}_{\mathrm{F}} = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} \\ M_{2,1} & M_{2,2} & M_{2,3} \\ M_{3,1} & M_{3,2} & M_{3,3} \end{bmatrix},$$

where

$$M_{1,1}|_{i+1+(j-1)(N+1)+(k-1)N(N+1),\ l+1+(m-1)(N+1)+(k-1)N(N+1)}$$
$$= \int_{\Omega_{\mathrm{ref}}} \sqrt{g}\left(g^{2,2}g^{3,3} - g^{2,3}g^{3,2}\right) \mathrm{lee}^{i,j,k}\mathrm{lee}^{l,m,n}\mathrm{d}\Omega, \quad i,l \in \{0,1,\cdots,N\},\ j,k,m,n \in \{1,2,\cdots,N\},$$

$$M_{2,2}|_{i+jN+(k-1)N(N+1),\ l+mN+(n-1)N(N+1)}$$
$$= \int_{\Omega_{\mathrm{ref}}} \sqrt{g}\left(g^{3,3}g^{1,1} - g^{3,1}g^{1,3}\right) \mathrm{ele}^{i,j,k}\mathrm{ele}^{l,m,n}\mathrm{d}\Omega, \quad j,m \in \{0,1,\cdots,N\},\ i,k,l,n \in \{1,2,\cdots,N\},$$

$$M_{3,3}|_{i+(j-1)N+kN^2,\ l+(m-1)N+nN^2}$$
$$= \int_{\Omega_{\mathrm{ref}}} \sqrt{g}\left(g^{1,1}g^{2,2} - g^{1,2}g^{2,1}\right) \mathrm{eel}^{i,j,k}\mathrm{eel}^{l,m,n}\mathrm{d}\Omega, \quad k,n \in \{0,1,\cdots,N\},\ i,j,l,m \in \{1,2,\cdots,N\},$$

$$M_{1,2}|_{i+1+(j-1)(N+1)+(k-1)N(N+1),\ l+mN+(n-1)N(N+1)}$$
$$= \int_{\Omega_{\mathrm{ref}}} \sqrt{g}\left(g^{2,3}g^{3,1} - g^{2,1}g^{3,3}\right) \mathrm{lee}^{i,j,k}\mathrm{ele}^{l,m,n}\mathrm{d}\Omega, \quad i,m \in \{0,1,\cdots,N\},\ j,k,l,n \in \{1,2,\cdots,N\},$$

$$M_{1,3}|_{i+1+(j-1)(N+1)+(k-1)N(N+1),\ l+(m-1)N+nN^2}$$
$$= \int_{\Omega_{\mathrm{ref}}} \sqrt{g}\left(g^{2,1}g^{3,2} - g^{2,2}g^{3,1}\right) \mathrm{lee}^{i,j,k}\mathrm{eel}^{l,m,n}\mathrm{d}\Omega, \quad i,n \in \{0,1,\cdots,N\},\ j,k,l,m \in \{1,2,\cdots,N\},$$

$$M_{2,3}|_{i+jN+(k-1)N(N+1),\ l+(m-1)N+nN^2}$$
$$= \int_{\Omega_{\mathrm{ref}}} \sqrt{g}\left(g^{3,1}g^{1,2} - g^{3,2}g^{1,1}\right) \mathrm{ele}^{i,j,k}\mathrm{eel}^{l,m,n}\mathrm{d}\Omega, \quad j,n \in \{0,1,\cdots,N\},\ i,k,l,m \in \{1,2,\cdots,N\},$$

and $M_{2,1} = (M_{1,2})^{\mathsf{T}}$, $M_{3,1} = (M_{1,3})^{\mathsf{T}}$, $M_{3,2} = (M_{2,3})^{\mathsf{T}}$.

4 Mass matrix $\mathsf{M}_{\mathbf{V}}$

M_{V} is the mass matrix of space $\mathrm{VP}_{N-1}(\Omega)$, and its entries are

$$\mathsf{M}_{\mathrm{V}}|_{i+(j-1)N+(k-1)N^2,\ l+(m-1)N+(n-1)N^2}$$
$$= \int_{\Omega_{\mathrm{ref}}} \frac{1}{\sqrt{g}} \mathrm{eee}^{i,j,k}\mathrm{eee}^{l,m,n}\mathrm{d}\Omega, \quad i,j,k,l,m,n \in \{1,2,\cdots,N\}.$$